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Wittgenstein on Mathematics and Certainties

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Abstract

This paper aims to contribute to the debate over epistemic versus non-epistemic readings of the 'hinges' in Wittgenstein's *On Certainty*. I follow Marie McGinn's and Daniele Moyal-Sharrock's lead in developing an analogy between mathematical sentences and certainties, and using the former as a model for the latter. However, I disagree with McGinn's and Moyal-Sharrock's interpretations concerning Wittgenstein's views of both relata. I argue that mathematical sentences as well as certainties are true and are propositions; that some of them can be epistemically justified; that in some senses they are not prior to empirical knowledge; that they are not ineffable; and that their primary function is epistemic as much as it is semantic.

Keywords

Wittgenstein – Marie McGinn – Daniele Moyal-Sharrock – mathematics – certainties

. . .

> I'll teach you differences.
>
> SHAKESPEARE, *KING LEAR, ACT 1, SCENE 4*

. . .

> To resolve these philosophical problems one has to compare things which it has never seriously occurred to any-one to compare.
>
> WITTGENSTEIN 1978: VII 15

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1 Introduction

This paper aims to contribute to the debate over epistemic versus non-epistemic readings of the 'certainties' (also known as 'hinges' or 'hinge propositions') of Wittgenstein's last notebooks, posthumously published as *On Certainty* (OC) in 1969. Interpreters on both sides of this debate have sometimes connected it to a further discussion: to wit, the discussion over the correct rendering of Wittgenstein's views on mathematical sentences. For instance, Marie McGinn (1989) and Danièle Moyal-Sharrock (2005) both defend their non-epistemic readings of the certainties in part by suggesting a parallel non-epistemic interpretation of Wittgenstein on mathematical sentences. According to McGinn and Moyal-Sharrock, Wittgenstein develops an analogy between mathematical sentences and certainties, and he uses the former as a model for the latter.

I agree with McGinn and Moyal-Sharrock that there are crucial parallels between Wittgenstein's treatment of certainties and his account of mathematical propositions. Nevertheless, I disagree with these two distinguished interpreters concerning Wittgenstein's views of both *relata*. In order to bring out the disagreement as clearly as possible, it is imperative to formulate the central tenets of epistemic and non-epistemic views of (Wittgenstein on) mathematical sentences and certainties. According to the non-epistemic position, mathematical sentences as well as certainties:

(1) are true only in an entirely empty sense and thus are not propositions;
(2) they cannot be epistemically justified;
(3) they are prior to empirical knowledge;
(4) they are ineffable, that is, they cannot be uttered in a meaningful way; and
(5) their primary function is semantic: they define the meaning of terms.

Naturally, as a summary of McGinn's and Moyal-Sharrock's sophisticated positions this five-part characterization is too coarse and insufficiently sensitive to important differences between the two interpreters. I will offer a more nuanced picture as I go along.

In any case, in what follows I shall try to argue that the non-epistemic view of mathematical sentences and certainties is not forced upon us by the evidence of Wittgenstein's writings. And I shall develop an alternative—'epistemic'—rendering point by point. Thus, mathematical sentences as well as certainties:

(1*) are true and are propositions;
(2*) some of them can be epistemically justified;

(3*) in some senses they are not prior to empirical knowledge;
(4*) they are not ineffable; and
(5*) their primary function is epistemic as much as it is semantic.

Duncan Pritchard (2007), Michael Williams (2004a, 2004b, 2007), and Crispin Wright (2004) have also defended versions of the epistemic view, and I am indebted to their work. I go beyond them in paying attention to the details of Wittgenstein's text and in engaging in greater detail with the non-epistemic view.

Here is a roadmap of what is to follow. I shall first go over some of the textual support for and against (1) to (5) for mathematical sentences and then discuss the respective evidence for and against (1) to (5) in the case of certainties. My main emphasis will be on the latter issue. As far as Wittgenstein on mathematics goes, I offer no more than impressions backed up by a modicum of textual evidence. A detailed interpretation of Wittgenstein's philosophy of mathematics, and a careful consideration of the vast secondary literature concerning it, lies beyond the scope of this paper.

2 The Nature and Goals of Wittgenstein's Reflections on the Philosophy of Mathematics

As a prolegomenon to my discussion of the content of Wittgenstein's thoughts on mathematics I want to briefly draw attention on their peculiarly hesitant tone and overall aims. McGinn and Moyal-Sharrock, as well as many other interpreters, ascribe to Wittgenstein definite, bold and radical views on a wide range of issues in the philosophy of mathematics. I am not convinced that such ascriptions harmonize with the following salient features of his writings.

First of all, Wittgenstein's texts are full of expressions of hesitation and doubt, and often these expressions concern the very bold and radical views often attributed to him (1978: I 107; 2000: 106, 121–122, 127, 166, 200, 30v, 31v–32r, 52v, 74v). Examples where Wittgenstein expresses uncertainty about the very radical views sometimes ascribed to him include the following:

> Isn't it odd to say: the formula "25 × 25 = 625" is the mark of a concept? And yet something tempts me to say that. Is that just non-sense or haste? … It must be in part an illness. (2000: 122, 74v–75r)

> I do not understand the human activities, the techniques of word-use, of mathematical sentences, of proofs. (2000: 117, 220–221)

> Thus it is as if the proof did not determine the sense of the proposition proved; and yet as if it did determine it. (1978: VI 10)

Note also that some of Wittgenstein's general and programmatic statements concerning his goals do not fit with interpretations according to which he seeks to establish radically new theses in the philosophy of mathematics:

> I may occasionally produce new interpretations, not in order to suggest they are right, but in order to show that the old interpretation and the new interpretation are equally arbitrary. (1976: 14)

> The goal is a perspicuous comparative presentation of all of the applications, illustrations, and interpretations of the calculus. (2000: 116, 55)

> It is the method of philosophy to listen to *all* voices and to reconcile them with one another. (2000: 109, 159)

It is not easy to suggest an overall interpretational strategy for doing justice to these features of Wittgenstein's texts. And I certainly do not wish to bar all attempts to work out what Wittgenstein thought about important issues in the philosophy of mathematics. In this paper I seek to do justice to these features in a modest fashion: I seek to identify passages which do not fit with radical, bold and radical views emphasised by McGinn and Moyal-Sharrock.

3 Mathematical Sentences, Propositions, and the Principle of Bipolarity

According to Moyal-Sharrock, the following train of thought leads Wittgenstein to deny that there are mathematical propositions. In order to express a proposition a sentence must be capable of being true, and capable of being false. (This is the 'principle of bipolarity.') But a mathematical sentence is not capable of being false: 2 + 2 = 5 is not false but meaningless. Ergo there are no mathematical propositions. (Moyal-Sharrock 2005: 36–38.) Moyal-Sharrock's evidence for this train of thought comes almost exclusively from texts of the early 1930s, such as Moore's report: Wittgenstein "sometimes said that [mathematical sentences] are not propositions at all… They are propositions of which the negation would be said to be 'impossible'" (Moore 1993: 72). Of later writings Moyal-Sharrock refers to only one passage in the *Remarks on the Foundations of Mathematics*: "There must be something wrong with our idea of the truth and falsity of our arithmetical propositions" (Wittgenstein 1978: I 135). Of course, Moyal-Sharrock also reminds us that Wittgenstein often speaks of mathematical sentences as functioning as rules (2007: 39–43).

McGinn writes that the question whether mathematical propositions are true or false "is, at bottom, entirely empty…calling them 'true' does not add anything to the fact of their being used, it is an honorific title" (1989: 128). In order to back up her reading she points to the following section from the *Remarks*: "The steps which are not brought into question are logical inferences. But the reason is not that they "certainly correspond to the truth" … —no, it is just that this is called 'thinking,' 'speaking,' 'inferring,' 'arguing'" (Wittgenstein 1978: I 156).

Considerations such as these do not seem to me to support the thesis that for the later Wittgenstein there are no mathematical propositions. To begin with, in lectures given in 1939 in English, Wittgenstein seems entirely comfortable with speaking of mathematical 'propositions' rather than 'sentences'—and even when stressing their uses as rules. For instance: "One might also put it crudely by saying that mathematical propositions containing a certain symbol are rules for the use of that symbol" (1976: 33).

Furthermore, throughout the 1930s and 40s Wittgenstein talks repeatedly of the *truth* of mathematical propositions, and without any obvious hesitation or reluctance:

> The proven mathematical proposition has, in its grammar, a preponderance towards truth. (2000: 113, 106v)

> …the proposition "25 × 25 = 625" may be true in two senses. If I calculate a weight with it… First, when used as a prediction of what something will weigh… In another sense, …if calculation shows this… (1976: 41)

> What a proof proves is that the proposition is true… (2000: 123, 66r)

> The truth of the proposition that 4 + 1 is 5 is thus, as it were, overdetermined…in that the one declares the result of the operation to be the criterion of the execution of the operation. (2000: 164, 48–49)

Moreover, in 1939 Wittgenstein is much less adamant than before that the negations of mathematical sentences are meaningless. In 1939 he seems more concerned to map our conflicting intuitions about the falsity or meaninglessness of expressions like "2 + 2 = 5":

> this "meaningless" road has now been trodden so often that it has become muddy… One can ask, How deep does his belief go? How far does

> he believe that 25x25 is 624? … Does he just say, “25x25 is 624”? Or does he go on to multiply it out? And if he does multiply it out, does he do the whole sum correctly except that he writes down the bottom line as “624” instead of “625”? And if so, what does he believe that’s wrong? One might say, in fact, that “He believes that 25 × 25 = 624” may correspond to many different states of affairs. (1976: 92; cf. 1978: I 107)

With respect to Wittgenstein’s claim that there is something wrong with our idea of the truth and falsity of arithmetical propositions, the important thing to note is that this claim only speaks against a *correspondence-theoretical* rendering of the truth of mathematical propositions. Indeed, sometimes Wittgenstein floats the redundancy theory as a serious alternative, or toys with the idea that to call a mathematical proposition ‘true’ is to sanction it for further use:

> Imagine the following queer possibility: we have always gone wrong up to now in multiplying 12x12. True, it is unintelligible how this can have happened, but it has happened. So everything worked out in this way is wrong! – But what does it matter? It does not matter at all! – And in that case there must be something wrong with our idea of the truth and falsity of our arithmetical propositions. (1978: I 135)

> But is a proof just constructing a proposition? Doesn’t it show also that the proposition is true? But that isn’t satisfactory. To say proposition p is true is just the same as to say p. (1976: 68)

> Should I say: the proof of p is the proof of its truth…? If we say [this] we are thinking: the proposition is now sanctioned, we can use it further… (2000: 121, 30v)

4 The Justification of Mathematical Sentences and Practices

McGinn claims that the certainty of logical and mathematical sentences is “a form of certainty for which the question of our justification for the judgements we accept is completely out of place” (1989: 139). Moyal-Sharrock agrees: “the rules of mathe-matical…languages are as ungrounded, arbitrary or unreasoned as those of chess” (2005: 40).

I find it difficult to square these proposals with the pivotal role Wittgenstein gives to proofs. After all, he seems to hold that proofs justify mathematical propositions, show that the latter are true, and give them their meaning:

> the proof always belongs to the grammar of that which has been proven. (2000: 122, 71v)

> What a proof proves is that the proposition is true… (2000: 123, 66r)

McGinn and Moyal-Sharrock might reply that although individual mathematical propositions or rules are justified in terms of proofs, mathematical practice *as a whole* is ungrounded. And there is definitely a sense in which this is true: there are no proofs for mathematical practice as a whole. But there is also a sense in which talk of the 'ungroundedness' of mathematical practice would be an overstatement. On the one hand, mathematical practice is in good part constituted by the proven mathematical propositions. And, on the other hand, although there cannot be proofs for mathematical practices, it is still possible to see them as rational responses to 'general facts' about us and about the world:

> "There are 60 seconds to a minute." This proposition is very *like* a mathematical one. …could we talk about minutes and hours, if we had no sense of time; if there were no clocks, or could be none for physical reasons; if there did not exist all the connexions that give our measures of time meaning and importance? In that case – we should say – the measure of time would have lost its meaning… (1978: VII 18)

Finally we cannot properly understand the issue of justification in the realm of mathematics without taking note of Wittgenstein's understanding of the relationship between experiment and proof. Wittgenstein thinks of a proof as an experiment that we have decided to treat as a norm for other experiments of the same kind: "We might have adopted 2 + 2 = 4 because two balls and two balls balances four. But now we adopt it, it is aloof from experiment—it is petrified" (1976: 98). That is to say, at first we have very strong and compelling empirical evidence for two times two balls equaling four balls in weight. But for this experiment to function as a proof we have to go further and immunize it against all possible empirical refutation. And we become persuaded or compelled (as Wittgenstein says) "for the greatest variety of reasons" to decide in favor of such immunization. One of these reasons is that the proof—in virtue of being a proof—gives the proven proposition a place in our system of mathematics:

> If the proof is a road taking us to this proposition, what role does the road play – once we have gone down it? It gives the proposition its place in a system. (2000: 122, 58v)

> The proof persuades me; ... I accept these transformations. – I let them determine my way of speaking. (Should I say: "for the greatest variety of reasons"?) (2000: 122, 58v–59r)

5 The Applications of Mathematics

McGinn's central thesis in this regard is that the "certainty concerning the propositions of mathematics and logic... [is] prior to knowledge of the results of the application of logical and mathematical techniques in empirical contexts..." (1989: 139) Moyal-Sharrock does not address this issue directly.

Taken in one way McGinn's contention is obviously true: what is applied is prior to the application. And yet, to leave matters here would be to miss some key features of Wittgenstein's position. I am thinking first and foremost of his occasionally-voiced suspicion that it is their application *outside of* mathematics that gives mathematical propositions their meaning: "It is the use outside mathematics, and so the meaning of the signs, that makes the sign-game into mathematics" (1978: V 2).

This idea does not contradict the earlier-mentioned view according to which it is a proof that gives meaning to mathematical propositions. There is no contradiction here since proof and application are intertwined and inseparable:

> We call these things proofs because of certain applications; and if we couldn't use them for predicting, couldn't apply them, etc., we wouldn't call them proofs. (1976, 38; cf. 2000: 124, 191)

The usefulness of the applications of a mathematical proposition plays a central role in convincing us to accept the proof of this proposition. And, the proof of a mathematical proposition (often? always?) gives the mathematical proposition its application.

6 The Ineffability of Mathematical Sentences

Moyal-Sharrock's provocative claim is that for Wittgenstein mathematical 'propositions' qua grammatical rules cannot be 'said': "They cannot be said *in* the language-game, for they support the language-game" (2005: 47). They exist only 'in action': "they can only *show* themselves *in* what we say and do" (2005: 72). McGinn does not make this strong a claim.

My first reply is that I cannot find good textual evidence for the ineffability interpretation. Of course, Wittgenstein does hold that mathematical practice has its roots in action rather than theorizing:

> The origin and the primitive form of the language game is a reaction; more complicated forms grow upon its basis. Language – I want to say – is a refinement, "im Anfang war die Tat." (2000: 119, 147)

And there are of course features and preconditions of our actions and practices that we do not—and are unable to—verbalize: "The essence of the language game is a practical method (a form of action), – no speculation, no idle talk" (2000: 188, 78v). And yet, all this does not mean that mathematical propositions themselves are ineffable or exist in action only.

Perhaps Moyal-Sharrock's point is not that *all* mathematical rules are ineffable but that *some* are. After all, with reference to OC 48—"out of a host of calculations certain ones might be designated as reliable once and for all, others as not yet fixed"—she distinguishes between mathematical sentences that are certainties and mathematical sentences that are not. Thus she believes that whereas "2 + 2 = 4" is a 'hinge' for her, "235 + 532 = 767" is not (2005: 102). So maybe it is only the mathematical 'certainties' that are ineffable.

Alas, I am still not convinced. *On Certainty* distinguishes between an individual's certainties regarding simple calculations, and the institutionalized or 'fossilized' certainty of mathematical sentences:

> It makes a difference: whether one is learning in school what is right and wrong in mathematics, or whether I myself say that I cannot be making a mistake in a proposition. (OC 664)

> The propositions of mathematics might be said to be fossilized. (OC 657).

Of course, some but not all calculations are certainties for Moyal-Sharrock. But mathematical sentences that have been immunized by mathematical proofs are certainties in a different sense, and they are certainties for us all: they have "as it were officially, been given the stamp of incontestability" (OC 655). And to be a mathematical sentence is to have a proof. Thus a certainty/non-certainty distinction does not seem to make sense with respect to mathematical sentences: all mathematical sentences are certainties. And hence the ineffability claim must hold for all or no mathematical sentences.

7 The Function of Mathematical Sentences

As already seen, McGinn and Moyal-Sharrock contend that mathematical ‘propositions’ are nothing but grammatical rules. This misses the fact that for Wittgenstein mathematical propositions also have important functions as epistemic rules (as emphasized by Wright 2004). Here is a toy example building on one of Wittgenstein’s own cases. Assume we have evidence for three propositions:

(a) Two apples have been put in the empty bowl.
(b) Two further apples have been put in the bowl.
(c) There are three apples in the bowl.

The mathematical proposition 2 + 2 = 4 as rule of evidence tells us that at least one of (a–c) must be wrong.

This is not to deny that mathematical sentences also have important functions as grammatical or semantic rules, and that there are deep and fundamental questions to be asked about the relationship between the two kinds of rules. At this point I am content with the reminder that Wittgenstein is perfectly open to the idea that mathematical rules have both kinds of functions. (I shall return to this issue below.)

This completes my all too brief objections to McGinn’s and Moyal-Sharrock’s views of Wittgenstein on mathematics. I hope to at least have raised some doubts in the reader’s mind concerning the adequacy of the non-epistemic reading of Wittgenstein’s views of mathematics. I now turn to the opposition between epistemic and non-epistemic readings concerning certainties or hinges.

8 Certainties, Propositions, and the Principle of Bipolarity

Are there hinge- or certainty-propositions and are they true or false? One reason for denying both questions might be the fact that Wittgenstein likens certainties to rules: “The propositions describing this world-picture…their role is like that of rules of a game…” (OC 95). There also are several places where Wittgenstein seems to explicitly deny that certainties are true or false. For instance:

> Giving grounds, however, justifying the evidence, comes to an end; – but the end are not certain propositions’ striking us immediately as true, i.e. it is not a kind of seeing on our part; it is our acting, which lies at the bottom of the language-game. (OC 204)

> If the true is what is grounded, then the ground is not true, not yet false. (OC 205)

Moyal-Sharrock contends that certainties or—as she prefers to say—'hinges,' "are divested of their propositional status inasmuch as their nature is similar to propositions of mathematics..." (2005: 39). Of course, if my observations in Sections 3–7 are anywhere near the mark, then this similarity speaks *against* the non-epistemic rendering of certainties, not for it.

Interestingly enough, Moyal-Sharrock acknowledges that the textual evidence in *On Certainty* does not unequivocally support her interpretation. She deals with passages in which certainties figure as propositions by speaking of "an ongoing...struggle [in the text] ...whose outcome—that our basic beliefs are nonpropositional—...is...best formulated in OC 204" (2005: 89).

McGinn's ideas on the propositional status of certainties are less clearly articulated. She write that the "technique-constituting role of these judgements [i.e., certainties] requires that their 'truth' is a matter of course, or better, that these judgements are not put up for question" (1989: 142).

I have a number of replies. I begin with a comment on Moyal-Sharrock's suggestion of a "struggle" in Wittgenstein's text. I have sympathies for this suggestion if it is meant as a reference to Wittgenstein's many expressions of hesitation or doubt about many allegedly central claims of *On Certainty* (cf. OC 35, 112, 321, 397, 400, 402, 405, 470, 532, 552, 567). However, I cannot find that Wittgenstein is in two minds about, or struggles with, the propositional account of certainties. I am also skeptical whether it makes sense to say that the outcome of an ongoing struggle in Wittgenstein's text should be formulated best by OC §204, when the struggle allegedly continues until OC 676.

More importantly still, note that there are numerous references to certainties as propositions, judgments, beliefs, or 'fundamental attitudes.' For instance (cf. also OC 10, 137, 140, 102, 144):

> The *truth* of certain empirical propositions belongs to our frame of reference. (OC 83)

> I want to say: We use judgments as principles of judgment. (OC 124)

> So it might be said: "The reasonable man believes: that the earth has been there since long before his birth, that his life has been spent on the surface of the earth, ..." (OC 327)

> I might therefore interrogate someone who said that the earth did not exist before his birth, in order to find out which of my convictions he was

> at odds with. And then it *might* be that he was contradicting my fundamental attitudes… (OC 238)

It is also noteworthy that *On Certainty* denies the existence of a 'sharp' boundary between rule and empirical proposition (OC 319) and that it reminds its readers that "the concept 'proposition' itself is not a sharp one" (OC 320). Such statements can be read as downplaying or abandoning the principle of bipolarity: we are permitted to speak of propositions even when the sentence in question functions as a rule and thus is not—at least on some of its uses—true or false.

In dealing with this strand of reflections in *On Certainty*, Moyal-Sharrock makes a surprising move (2005: 86). She first quotes OC 309: "Is it that rule and empirical proposition merge into one?," and then claims that OC 308 (i.e., the preceding) paragraph gives a 'negative answer': "I am inclined to believe that not everything that has the form of an empirical proposition is one." On Moyal-Sharrock's reading, in OC 308 Wittgenstein rejects the idea that—in the case of hinges—rule and empirical proposition merge into one and merely insists that not everything that looks like an empirical proposition actually is one. This interpretational move is awkward insofar as, on Moyal-Sharrock's rendering, the answer comes before the question. Correcting this maneuver, and sticking to the order of the two remarks, it seems more plausible to suggest the following: the merging of rule and empirical proposition is taken by Wittgenstein to be a possibility *even after* we have noted that not everything that has the form of an empirical proposition actually is one.

As far as the truth of certainties is concerned, Wittgenstein again only rejects a blanket application of the correspondence theory. He finds talk of "tallying with the fact" a misleading expression since what such 'tallying' amounts to is dependent upon context (OC 199). And yet, Wittgenstein does not refuse to speak to of certainties as true:

> "2 × 2 = 4" is a true proposition of arithmetic… (OC 10)

> The *truth* of certain empirical propositions belongs to our frame of reference. (OC 83)

> Supposing it wasn't true that the earth had already existed long before I was born – how should we imagine the mistake being discovered? (OC 301)

In other words, just like in his remarks on the philosophy of mathematics, in *On Certainty* Wittgenstein rejects a specific theory of truth, but he does not reject truth talk per se, not even with respect to certainties.

It is also worth stressing that at least later parts of *On Certainty* are ready to speak of certainties as things we know:

> I know, not just that the earth existed long before my birth, but also that it is a large body, that this has been established, that I and the rest of mankind have forebears, that there are books about all this, that such books don't lie, etc. etc. etc. And I know all this? I believe it. This body of knowledge has been handed on to me and I have no grounds for doubting it, but, on the contrary, all sorts of confirmation. And why shouldn't I say that I know all this? Isn't that what one does say?' (OC 288; cf. OC 340, 360, 379, 380, 431, 555, 558, 579, 613)

At this point the reader might object that I am neglecting passages in which Wittgenstein explicitly denies that certainties can be meaningfully said to be true. The most prominent passage obviously is OC 205: "If the true is what is grounded, then the ground is not true, not yet false." I remain unconvinced. OC 205 shows that certainties are not true only if the mentioned 'ground' are the certainties. And there are passages that suggest a different reading: the ultimate ground are facts about nature and our ways of acting (cf. OC 358, 359, 559). If that is true—and I shall return to this issue in a moment—then OC §205 does not speak against certainties being true.

9 The Justification of Certainties

McGinn and Moyal-Sharrock both insist that the certainty of hinge-propositions is not epistemic, and that regarding them there is "no space for the notion of grounds to squeeze in" (McGinn 1989: 157). I believe that this is correct for some types of certainties but not for others.[1] In making this claim I am taking seriously the second-to-last paragraph (674) of *On Certainty*. Here Wittgenstein first describes as his central interest as "cases in which I rightly say I cannot be making a mistake" and then goes on to note "I can enumerate various typical cases, but not give any common characteristic." Moyal-Sharrock's reponse to Wittgenstein's modesty is dismissive:

1 In addition to McGinn and Moyal Sharrock, Annalisa Coliva (2010) also maintains that, according to Wittgenstein, we do not bear an epistemic relation to certainties or hinges. Coliva finds three possible uses of "I know" and "knowledge" in *On Certainty*: one empirical, one grammatical, and one nonsensical. Coliva thinks that Wittgenstein has the grammatical use in mind when saying that certainties can be known. Coliva takes §58 to support her reading. I remain unconvinced, but the issue deserves a separate treatment elsewhere.

> No common characteristic. And yet, … I have listed precisely that: characteristics…common to all hinges… He did not refine his awareness of the multifariousness of hinges into the insight that this multifariousness does not prevent them from sharing the same features… (2005: 100)

According to Moyal-Sharrock these 'same features' are that all hinges are indubitable, foundational, non-empirical, grammatical, ineffable, and existing in action only.

I am with Wittgenstein, especially as concerns the question whether, and to what extent, certainties can be justified. But what then are the "various typical cases" of which Wittgenstein says he cannot "give any common characteristic"? I suggest that we can distinguish between ten types of cases, at least as far are the examples of *On Certainty* are concerned.

(i) Perceptual beliefs about close familiar medium-size objects.
E.g.: "I believe there is an armchair over there." (OC 193)

(ii) Memory beliefs of salient features of one's autobiography.
E.g.: "I have lived in this room for weeks past." (OC 416)

(iii) Beliefs based on simple deductive reasoning; e.g. calculations.
E.g.: "12 × 12 = 144" (OC 43)

(iv) Simple inductive beliefs, e.g. about familiar simple objects.
E.g.: "After putting a book in a drawer, I assume it is there" (OC 134)

(v) Testimonial beliefs based on parents' or textbook testimony.
E.g.: "textbooks of experimental physics… I trust them." (OC 600)

(vi) Simple plural-source beliefs:
semantic beliefs, e.g.: "My name is L.W." (OC 425)
general knowledge, e.g.: "Every human being has a brain." (OC 159)

(vii) Mathematical propositions.
E.g.: "officially, been given the stamp of incontestability." (OC 655)

(viii) Fundamental empirical-scientific beliefs.
E.g.: "The earth is round." (OC 291); "Water boils at 100°C." (OC 293)

(ix) Fundamental religious beliefs
E.g.: "Jesus only had a human mother." (OC 239)

(x) Beliefs that constitute domains-of-knowledge
E.g.: "the earth has existed for many years past" (OC 411); "the earth exists" (OC 209).

At this stage it is particularly pertinent to emphasize that (i) to (x) behave differently as far as epistemic justification is concerned. The first six behave similarly. The confidence with which we hold these beliefs is empirically based. As Wright correctly points out concerning your certainty "I have two hands": it is grounded in "your lifelong experience of yourself as handed" (2004: 36). In similar ways, Wittgenstein had a lifelong experience of being called 'Ludwig Wittgenstein'; he had a long-term experience of living in certain houses and apartments; he had a many-decades-long experience of being informed correctly by certain accredited textbooks; and he had a decades-long experience of being (regarded as) a reliable calculator or deductive and inductive reasoner. At the same time it cannot be emphasized enough, precisely because, in these cases, our evidence is so wide, so deep, so multifarious, and so temporarily extended, that most of it lies beyond our present point of view. That is why we cannot easily disclose this evidence to others. In other words, the relevant evidence is both overwhelming and yet, in a way, inaccessible. For most purposes it is thus dialectically mute. Nevertheless, our confidence in these beliefs is such that no ordinary evidence can generally either undermine or confirm them. It is these features that for Wittgenstein usually make it odd to self-ascribe these beliefs as 'knowledge.'

For a consideration of the case of mathematical propositions (vii), it is worth to reconsider Wittgenstein's earlier-quoted example of the four balls, and the 'experiment' according to which two balls and two balls balance four balls. We can repeat this experiment endlessly, and are likely to get the same result (almost) all of the time. The experimental evidence that two balls and another two balls balance four balls provides us with a very good reason for immunizing the empirical proposition into a mathematical proposition and thereby treating the experiment as a proof. Compared with (i) to (vi), the evidence for instances of (vii) is much less diffuse and intractable. Moreover, there is no direct analogue in cases (i) to (vi) for the 'greatest variety of reasons' that persuade us to immunize certain empirical propositions into mathematical rules. And finally, the process leading to the adoption of mathematical propositions is often accessible—at least to historical research—in a way that cases of (i) to (vi) are not.

As concerns (viii) fundamental scientific doctrines, Wittgenstein is happy to speak of knowledge:

> We know that the earth is round. We have definitively ascertained that it is round. We shall stick to this opinion, unless our whole way of seeing nature changes. (OC 291)

Surely, the "we have definitively ascertained" is a reference to strong evidence for the belief that the earth is round. This role of evidence is not cancelled by our decision not to accept contrary evidence—"We shall stick to this opinion"—as long as we stick to our current view of the world.

(viii) differs from (i)–(vi) in that the way in which we arrive at our certainty is communal, scientific and tractable. The evidence can be reviewed, and it is explicitly learnt by students and children. And (viii) differs from (vii), the case of mathematical propositions, in that the experiment-proof structure is less rigid.

Fundamental religious beliefs (ix) often get mentioned in *On Certainty* but for a more detailed analysis one must turn to Wittgenstein's "Lectures on Religious Belief" (1966). There Wittgenstein suggests that the evidence for religious beliefs is of a very different kind from ordinary evidence: "Reasons [for religious beliefs] look entirely different from normal reasons. They are, in a way, quite inconclusive" (1966: 56; cf. Kusch 2011).

Finally, beliefs that constitute domains-of-knowledge (x), like "The earth exists," behave very differently from mathematical and fundamental scientific doctrines. Tokens of this type do not fit the model of "evidentially strongly supported belief immunized into a certainty." After all, there is no non-circular way to provide evidence for the existence of the earth. (x) also differs from (i) to (vi) insofar as it would be odd to claim that our belief that the earth exists is grounded in a lifetime experience of the earth existing. This is not how we use the term 'experience' in ordinary life.

This then is my reply to McGinn's and Moyal-Sharrock's claim that certainties cannot be epistemically justified. The claim is only partially correct. It is fully true only for category (x). Evidential considerations play varying roles in the other categories.

At this stage it might be objected that in the above I have overlooked the strong textual evidence in *On Certainty* against the justifiability of certainties. Two passages seem particularly relevant here. OC 559 reads: "You must bear in mind that the language-game…is not based on grounds. It is not reasonable (or unreasonable)." Does this show that Wittgenstein regards all certainties as beyond epistemic justification? I think not. To see this, it is crucial to take proper account of the context of this passage. In §558 we were told that the behavior of water is 'fused into the foundations of our language-game' with water. Following on from this §559 is most naturally read as saying that in this case it is the physical facts underlying our language-game—rather than certainties—that are neither reasonable nor unreasonable. Of course that the physical facts underlying our language-game are neither reasonable nor unreasonable says nothing about the evidence we might have for certainties.

The second important passage is OC 358–359:

> Now I would like to regard this certainty…as a form of life. … But that means I want to conceive it as something that lies beyond being justified or unjustified; as it were, as something animal.

Before we draw the conclusion that all certainties are beyond justification, we had better again pay attention to the example that is being discussed here. This is a comment on the certainty with which Wittgenstein visually identifies at close range his chair as a chair (OC 355). And this kind of certainty is normally beyond (the need for further) justification: it is in our 'animal' nature to blindly trust our senses in such a case. But remember: this is just one of ten types of certainties.

10 Are Certainties Prior to Knowledge?

McGinn holds that the certainties constitute "descriptive techniques" and that the techniques in turn are essential for the possibility of knowledge (1989: 144–145). And Moyal-Sharrock insists that "[w]e do not *know* the primitive beliefs that underpin our knowledge" and admits that what she offers on the relationship between certainties and knowledge is an 'unquestionably foundational story" (2005: 78–79).

Wittgenstein does indeed think that all inquiry presupposes some certainties:

> That is to say, the *questions* that we raise and our *doubts* depend on the fact that some propositions are exempt from doubt, are as it were like hinges on which those turn. OC 341; cf. OC 96

This thought must be balanced, however, against other ideas. On the one hand, and as seen in the last section, at least some certainties originate from justified beliefs or knowledge. And, on the other hand, certainties and items of knowledge presuppose, constrain and support one another in a structure of mutual epistemic support:

> It is not single axioms that strike me as obvious, it is a system in which consequences and premises give one another *mutual* support. (OC 142)

> I have arrived at the rock bottom of my convictions. And one might almost say that these foundation-walls are carried by the whole house. (OC 248)

11 Are Certainties Ineffable and 'In Action' Only?

Concerning this question Moyal-Sharrock is more outspoken than McGinn. She is adamant that hinges 'cannot be said' and that they exist 'in action' only (2005: 47, 72). The following passages give some initial support to her interpretation:

> Someone says irrelevantly: "That's a tree." … Shouldn't I be at liberty to assume he doesn't know what he is saying…? (OC 468)

> Giving grounds…comes to an end; but the end are not certain propositions striking us immediately as true; …it is our *acting*, which lies at the bottom of our language-game. (OC 204)

I have two comments. The first focuses on differences between the ten categories of certainties. Wittgenstein does indeed hold that there are contexts in which it does not make sense to verbally express true perceptual beliefs about close familiar medium-size objects. Think of contexts in which the addressee of the perceptual report cannot fail to perceive the very same objects, and where the speaker is aware of this fact. And yet, and as already seen, the same is not true for mathematical propositions. Although mathematical propositions are certainties, it does make good sense to report them to others. Nor does the ineffability claim work for fundamental scientific doctrines: so far from becoming ineffable, they often turn into proverbs or platitudes (cf. Shapin 2001). And something similar is obviously true for religious certainties: these are typically repeated again and in again in prayers.

My second comment concerns the alleged textual evidence for ineffability, and especially OC 204. The context of this comment is the certainty that "the earth already existed long before my birth" (OC 203). As earlier mentioned, this is a certainty for which we cannot possibly assemble evidence—any attempt to do so would have to presuppose the very proposition at issue. This is crucial with respect to OC 204: "The earth has existed since long before my birth" is not a certainty for us because we have come to appreciate—to see—its immediate truth or its evidence. It is a certainty for us because our actions move "always already" in a temporal horizon that essentially includes the past.

12 Functions for Certainties

Moyal-Sharrock renders hinges as grammatical rules—one important subset of which are linguistic hinges, "that precisely define our use of individual words and of numbers" (2005: 102). And McGinn writes: "Moore-type propositions… show how the words of our language are used; they show us what a "hand" is, what "the world" is" (1989: 142).

McGinn and Moyal-Sharrock are right to stress this grammatical/linguistic role of certainties. But they pay too little attention to the various epistemic roles that certainties do also play. On this point I am with Wright (2004). The link between the semantic and the epistemic is clear for instance in the following passage:

> When a child learns language it learns at the same time what is to be investigated and what not. When it learns that there is a cupboard in the room, it isn't taught to doubt whether what it sees later on is still a cupboard or only a kind of stage set. (OC 472)

To throw the epistemic roles of certainties into sharper relief, it is best to return to my earlier distinction between ten types of cases in which "I rightly say I cannot be making a mistake" (OC 674). Above I went through these categories in order to show that the question whether certainties are supportable by evidence does not have a single answer; some types of certainties are supportable in this way, others are not. At this stage I want to focus on the role of different types of certainties as rules of evidence or general epistemic commitments.

We can again discuss categories (i) to (vi) together: (i) perceptual beliefs about close familiar medium-size objects, (ii) memory beliefs of salient features of one's autobiography, (iii) beliefs based on simple deductive reasoning, (iv) simple inductive beliefs, for instance. about familiar simple objects, (v) testimonial beliefs based on parents' or textbook testimony, and (vi) simple plural-source beliefs: such as semantic beliefs or general knowledge. The 'epistemic rulishness' of these propositions consists in our prioritising their evidence over contrary evidence: for instance, my evidence for my perceptual belief about close familiar medium-size objects over your testimony; my evidence for the reliability of standard textbooks over the report of a layperson; my evidence for my reliability as a calculator over your divergent calculation; etc. Wright (2004: 37) rightly stresses that this prioritising is not itself justified by experience, at least not directly. Treating one kind of evidence as superior to another kind of evidence is part of the 'logic of a language game': "What counts as an adequate test of a statement belongs to logic. It belongs to the description of the language-game" (OC 82).[2]

For mathematical propositions (vii) I have already given an example of how they function as evidential rules. This too is case where one kind of evidence is prioritised over another: the evidence for the mathematical proposition is prioritised over empirical evidence (a) to (c).

As far as fundamental scientific doctrines (viii) are concerned, the prioritising of one kind of evidence over others goes together with marking the boundaries of scientific practice, and being reference points in controversy:

> It is clear that our empirical propositions do not all have the same status, since one can lay down such a proposition and turn it from an empirical proposition into a norm of description. Think of chemical investigations. Lavoisier makes experiments with substances in his laboratory and now he concludes that this and that takes place when there is burning. He does not say that it might happen otherwise another time. (OC 167)

Regarding fundamental religious beliefs (ix), the "Lectures on Religious Belief" emphasize that that such beliefs can outweigh even scientific evidence (Wittgenstein 1966: 56).

Finally, beliefs that constitute domains-of-knowledge (x) behave very differently from mathematical and fundamental scientific doctrines. As already noted, these beliefs do not fit the model of 'evidentially strongly supported belief immunized into a certainty.' Insofar as type (x) beliefs constitute domains of knowledge, they rule out epistemic-sceptical scenarios. And, intriguingly enough, given the overall perspective of this paper, in explaining the anti-sceptical role of domain-constituting beliefs, *On Certainty* draws again on analogies with mathematics. Two lines of thought can be distinguished.

The first centres on the incoherence of global error:

> So is the *hypothesis* possible, that all the things around us don't exist? Would that not be like the hypothesis of our having miscalculated in all our calculations? (OC 55)

If "all the things around us exist" is false, then all our empirical beliefs are false. And to claim that all our empirical beliefs are false is analogous to claiming that all our calculations are false. The latter does not make sense: our common ways of calculating determine what our mathematical system is, and

2 Coliva (2010) argues that the term "logic" in the later Wittgenstein means (roughly) grammar, and that what belongs to logic plays a rule-like role.

what counts as true or false within it. Hence there cannot be the gap between our common ways of calculating (on the one hand) and mathematical truth (on the other hand) that the idea of global mathematical error presupposes.

The analogy for global empirical error is not spelled out—it seems to rely on a form of anti-realism. Our epistemic folkways determine what our epistemic system is, and what counts as justified or unjustified, true or false, within it. Hence there cannot be the gap between our epistemic folkways and truth that the idea of global external-world scepticism presupposes. This involves the idea that truth is what you get if you follow certain procedures. Truth is not something against which your system of procedures can be measured. *Pace* Wright (2004), there thus is an important anti-sceptical strand in *On Certainty* that builds on a form of 'internal realism.'

The second anti-skeptical train of thought involving mathematics surfaces in OC 375 and 392:

> Here one must realize that complete absence of doubt at some point, even where we would say that "legitimate" doubt can exist, need not falsify a language-game. For there is also something like *another* arithmetic. I believe that this admission must underlie any understanding of logic. (OC 375)

> What I need to shew is that a doubt is not necessary even when it is possible. That the possibility of the language-game doesn't depend on everything being doubted that can be doubted. (This is connected with the role of contradiction in mathematics.) (OC 392)

The epistemic sceptic who seeks to convince us that our epistemic system is flawed unless it is able to guarantee that we will not have false beliefs in sceptical scenarios is like the philosopher or mathematician who thinks that a logical or mathematical system is worth nothing without a proof of its consistency. Of course, Wittgenstein famously rejects the contention that a logical or mathematical system is worth nothing without a proof of its consistency, sometimes on the basis of the further comparison with regimes of measurement: a formal system that allows for the derivation of contradictory statements is like a regime of measurement employing stretchable rulers. And for Wittgenstein there is no absolute viewpoint from which one system of measurement is to be preferred over another. "To manufacture rulers out of ever harder, more unalterable material" is a matter of choice: "Certainly it is right; if that is what one wants!" (1978: VII 15).

> "But a contradiction in mathematics is incompatible with its application. … Its effect is e.g. that of non-rigid rulers which permit various results

> of measuring by being expanded and contracted." But was measuring by pacing out not measuring at all? And if people worked with rulers of dough, would that of itself have to be a called wrong? (1978: VII 15)

Application to epistemic scepticism: Like mathematical systems so also epistemic systems develop and are chosen 'for the greatest variety of reasons.' The strict and demanding epistemic system the sceptic insists on is not obligatory or uniquely accurate. Whether a measurement is accurate or inaccurate depends on the purposes of the practice in which is the measurement is to be used. The skeptic is like the metrologist who says that unless you can prove that your rulers are infinitely rigid, all your measurements are suspect:

> If someone wanted to arouse doubts in me and spoke like this: here your memory is deceiving you, there you've been taken in, there again you have not been thorough enough in satisfying yourself, etc., and if I did not allow myself to be shaken but kept to my certainty – then my doing so cannot be wrong, even if only because this is just what defines a game. (OC 497)

13 Conclusion

McGinn and Moyal-Sharrock are right: there are important parallels between (some of) Wittgenstein's views on mathematical sentences and central aspects of his account of certainties. However, as I have tried to gesture in the above, these parallels do not unequivocally support a non-epistemic rendering of both *relata*. There also are strands of reflections in Wittgenstein's text that justify an epistemic interpretation of both mathematical sentences and certainties. And it may sometimes be doubted whether the non-epistemic glosses that McGinn and Moyal-Sharrock put on key passages are adequate to the wider contexts in which these passages appear.

Of course this paper does not 'decisively refute' McGinn's and Moyal-Sharrock's readings. This is a short paper, and the issues are intricate and complex. But I hope to have done enough here to suggest the fruitfulness of a debate over particular text paragraphs and contexts in Wittgenstein's oeuvre in order to establish the respective strengths and weaknesses of epistemic and non-epistemic readings.[3]

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